Distributed Estimation:

LEAST SQUARES:

$$\underset{\sim}{z} = \underset{\sim}{H}\,\theta + v$$

parameters

noise

$$\underbrace{\begin{bmatrix} z_1 \\ \vdots \\ z_p \end{bmatrix}}_{\text{dep. variable}} = \underbrace{\begin{bmatrix} H_{11} & \cdots & H_{1q} \\ & & \\ & & \end{bmatrix}}_{\text{ind variables}} \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_q \end{bmatrix} + \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} V_i$$

$\leftarrow$ data pts $\rightarrow$ $\leftarrow p \rightarrow$

$$v_i \sim N(0, V_i)$$

$$H \in \mathbb{R}^{p \times q}$$

Cost: $J(\theta) = (z - H\theta)^T V^{-1} (z - H\theta)$

$$\hat{\theta} = \underset{\theta}{\mathrm{argmin}} \{ J(\theta) \} \Rightarrow \hat{\theta} = (H^T V^{-1} H)^{-1} H^T V^{-1} z$$

Distributed

$$z_1 = H_1 \theta + v_1$$
$$\vdots$$
$$z_n = H_n \theta + v_n$$

$$V = \begin{bmatrix} V_1 & & V_{1n} \\ & \ddots & \\ 0 & & V_n \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} H_1 \\ \vdots \\ H_n \end{bmatrix} \theta + \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$$\hat{\theta} = \left[\left[H_1^T \cdots H_n^T \right] \begin{bmatrix} V_1^{-1} & & 0 \\ & \ddots & \\ 0 & & V_n^{-1} \end{bmatrix} \begin{bmatrix} H_1 \\ \vdots \\ H_n \end{bmatrix} \right]^{-1} \left[H_1^T \cdots H_n^T \right] \begin{bmatrix} V_1^{-1} & & 0 \\ & \ddots & \\ 0 & & V_n^{-1} \end{bmatrix} \begin{bmatrix} z_1 \\ \vdots \\ z_n \end{bmatrix}$$

$$= \Big(\underbrace{\sum_i H_i^T V_i^{-1} H_i}_{P_i} \Big)^{-1} \underbrace{\sum_i H_i^T V_i^{-1} z_i}_{\tau_i}$$

Ea node: tracks $P_i, \Sigma_i \longrightarrow$ consensus on these

Covariances:

$v_i \in \mathbb{R}^n \qquad v_i \sim N(0, V_i)$

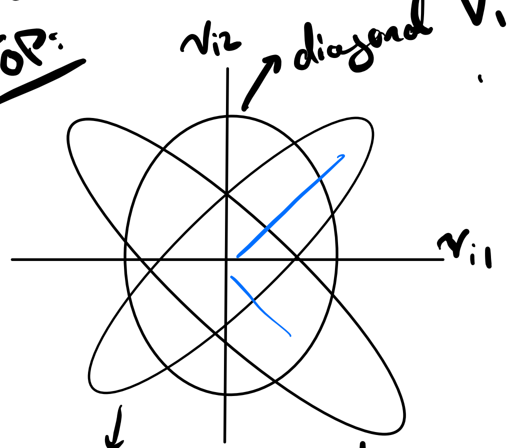
$$V_i = E[v_i v_i^T] = \int_{\mathbb{R}^n} v_i(x) v_i(x)^T p(x)\,dx$$

$n \times 1 \quad [\;n\;]$

Normal Distribution $\quad p(v_i) = \sim e^{-\frac{1}{2}(v_i^T V_i^{-1} v_i)}$

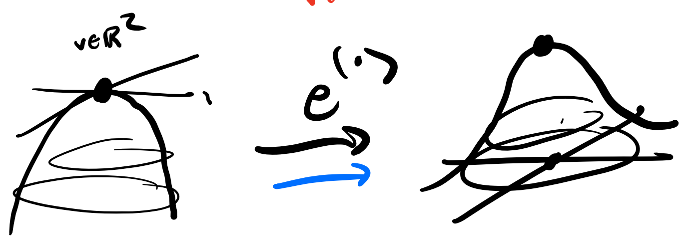
PD matrix

TOP:



positive correlation v_{i1} & v_{i2}

negative correlation v_{i1} & v_{i2}



$-\frac{1}{2} v_i^T V_i^{-1} v_i$

Kalman Filter: $\theta \rightarrow x$

Dynamics

$$x(k+1) = A(k)x(k) + B(k)u(k) + w(k) \quad \text{in general}$$

$$x(k+1) = Ax(k) + w(k) \leftarrow \text{for now.}$$

$$w(k) \sim N(0, W_k)$$

Measurement

$$z(k) = H(k)x(k) + v(k)$$

Before for LS: H was tall.

now $H(k)$ usually fat.

assume $w(k) = 0$

$$z(0) = H(0)x(0) + v(0)$$
$$z(1) = H(1)x(1) + v(1)$$
$$\vdots$$
$$z(k) = H(k)x(k) + v(k)$$

$$\begin{bmatrix} z(0) \\ z(1) \\ \vdots \\ z(k) \end{bmatrix} = \begin{bmatrix} H(0)x(0) \\ H(1)Ax(0) \\ \vdots \\ H(k)A^{k-1}x(0) \end{bmatrix} + \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(k) \end{bmatrix}$$

$$\begin{bmatrix} z(0) \\ \vdots \\ z(k) \end{bmatrix} = \begin{bmatrix} H(0) \\ H(1)A \\ \vdots \\ H(k)A^{k-1} \end{bmatrix} x(0) + \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(k) \end{bmatrix}$$

full column
matrix =
observability

Notation:

x: true state.

$\hat{x}$: state estimate:

$\hat{x}(k|k-1)$ before take a meas. at time k

$\hat{x}(k) = \hat{x}(k|k)$ after measurement.

Σ: error covariance

$\tilde{x}(k|k-1) = \hat{x}(k|k-1) - x(k)$

$\tilde{x}(k) = \hat{x}(k) - x(k)$

$\Sigma(k) = E[\tilde{x}(k)\tilde{x}(k)^T]$

$\Sigma(k|k-1) = E[\tilde{x}(k|k-1)\tilde{x}(k|k-1)^T]$