	Problem					Necessary Conditions			
	System	Cost	Time	Target Set	Hamiltonian	(1)	(2)	(3)	(4)
1	$\dot{x} = f(x,u)$	$L = L(x,u)$, $K = 0$	t_f Free	x_f Fixed end point	$H(x,u,\lambda) = L(x,u) + \lambda^T f(x,u)$	$\dot{x}^* = \nabla_\lambda H\|_*$ $\dot{\lambda}^* = -\nabla_x H\|_*$	$H(x^*,u^*,\lambda^*) = \min_{u\in U} H(x^*,u,\lambda^*)$ Or $H(x^*,u^*,\lambda^*) \le H(x^*,u,\lambda^*)$, $\forall u \in U$	$H^*(t) = H^*(t_f) = 0$	No condition on $\lambda^*(t_f)$
2				S_f k-fold in $\Re^n$ $g_i(x) = 0$, $i = 1,2,\cdots,n-k$					$\lambda^*(t_f)$ Normal to S_f at $x^*(t_f)$ $\lambda^*(t_f) = \sum_{i=1}^{n-k} \alpha_i \nabla_x g_i\|_{x^*(t_f)}$
3		$L = L(x,u)$, $K = K(x)$		$\Re^n$ Free end point					$\lambda^*(t_f) = \nabla_x K\|_{x^*(t_f)}$
4		$L = L(x,u)$, $K = 0$	t_f Fixed	x_f Fixed end point				$H^*(t) = H^*(t_f) = \text{constant}$	No condition on $\lambda^*(t_f)$
5		$L = L(x,u)$, $K = K(x)$		S_f k-fold in $\Re^n$ $g_i(x) = 0$, $i = 1,2,\cdots,n-k$					$\lambda^*(t_f) = \sum_{i=1}^{n-k} \alpha_i \nabla_x g_i\|_{x^*(t_f)} + \nabla_x K\|_{x^*(t_f)}$
6	$\dot{x} = f(x,u,t)$	$L = L(x,u,t)$, $K = 0$	t_f Free	x_f Fixed end point	$H(x,u,\lambda,t) = L(x,u,t) + \lambda^T f(x,u,t)$	$\dot{x}^* = \nabla_\lambda H\|_*$ $\dot{\lambda}^* = -\nabla_x H\|_*$	$H(x^*,u^*,\lambda^*,t) = \min_{u\in U} H(x^*,u,\lambda^*,t)$ Or $H(x^*,u^*,\lambda^*,t) \le H(x^*,u,\lambda^*,t)$, $\forall u \in U$	$H^*(t) = -\int_t^{t_f} (\nabla_t H\|_*) d\tau$ $H^*(t_f) = 0$	No condition on $\lambda^*(t_f)$
7				$g(t)$ Moving point				$H^*(t) = \lambda^{*T}(t_f)(\frac{dg}{dt}\|_{*,t_f}) - \int_t^{t_f} (\nabla_t H\|_*) d\tau$ $H^*(t_f) = \lambda^{*T}(t_f)(\frac{dg}{dt}\|_{*,t_f})$	
8		$L = L(x,u,t)$, $K = K(x,t)$		S_f (k+1)-fold in $\Re^n \times (T_1,T_2)$ $g_i(x,t) = 0$, $i = 1,2,\cdots,n-k$				$H^*(t) = H^*(t_f) - \int_t^{t_f} (\nabla_t H\|_* + \nabla_t^2 K\|_*) d\tau$ $H^*(t_f) = \sum_{i=1}^{n-k} \alpha_i \nabla_t g_i\|_{*,t_i} - \nabla_t K\|_{*,t_f}$	$\lambda^*(t_f) = \sum_{i=1}^{n-k} \alpha_i \nabla_x g_i\|_{x^*(t_f)} + \nabla_x K\|_{x^*(t_f)}$
9				$\Re^n$ Free end point				$H^*(t) = H^*(t_f) - \int_t^{t_f} (\nabla_t H\|_* + \nabla_t^2 K\|_*) d\tau$ $H^*(t_f) = -\nabla_t K\|_{*,t_f}$	$\lambda^*(t_f) = \nabla_x K\|_{x^*(t_f)}$
10		$L = L(x,u,t)$, $K = 0$	t_f Fixed	x_f Fixed end point				$H^*(t) = H^*(t_f) - \int_t^{t_f} (\nabla_t H\|_*) d\tau$	No condition on $\lambda^*(t_f)$
11				S_f k-fold in $\Re^n$ $g_i(x) = 0$, $i = 1,2,\cdots,n-k$					$\lambda^*(t_f)$ Normal to S_f at $x^*(t_f)$ $\lambda^*(t_f) = \sum_{i=1}^{n-k} \alpha_i \nabla_x g_i\|_{x^*(t_f)}$
12		$L = L(x,u,t)$, $K = K(x)$		$\Re^n$ Free end point					$\lambda^*(t_f) = \nabla_x K\|_{x^*(t_f)}$

This table of Pontryagin's Minimum Principle was disseminated by Prof. Marwan Simaan as part of the lecture notes for the optimal control course (Ref. M. Athans and P. Falb, "Optimal Control", Lincoln Lab., 1966, pp. 306-307). Note, the Hamiltonian is defined as H = < λ , f > + L , hence the "minimum" principle.