Modeling and Decision-Making in Complex Dynamical Systems of Systems with One Opposing Subsystem

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***Abstract*—Many large enterprises consist of a large number of interacting subsystems that operate harmoniously and make decisions that are designed for the benefit of the entire enterprise. When one subsystem has been attacked and is made to oppose and operate in a manner that is confrontational with the others, the entire system suffers, resulting in an adversarial environment. Such an environment may affect not only the decision-making process of the attacked subsystem but also possibly the other remaining subsystems as well. The disruption caused by one dissenting subsystem will cause the remaining subsystems to either coalesce as a team and act as a single team-based decision-maker, or disintegrate and act as independent decision-making entities. The decision-making process in these types of complex systems of systems is best analyzed within the general framework of game theory. In this paper, we will develop an analysis that provides a theoretical basis for the decision-making process of such complex systems. We show how cooperation can produce non-inferior Pareto-optimal type strategies that are fair and acceptable to all subsystems while at the same time yield strategies that are able to confront and minimize the damage caused by the opposing subsystem. Similarly, we examine the process when the team decides to let the decision maker of one subsystem make decisions on behalf of the entire team. An example of a system consisting of three subsystems is included to illustrate the results.**

I. INTRODUCTION

In the coming decade, large enterprises will consist of massive systems consisting of machines, human devices or virtual subsystems all interconnected through crucial technologies such as the Internet of Things (IoT) [1]. The IoT typically encompass a network of uniquely identifiable interconnected virtual and physical objects, such as sensors, radio frequency identification tags, actuators or mobile agents, which are able to communicate and exchange data with each other to perform different tasks [2]. These complex enterprises are referred to as System of Systems (SoS). Given their large scale and the heterogeneity of their environment, such SoSs are more vulnerable to security threats than other segregated systems. Thus, security requirements in these systems will be more stringent than those of conventional isolated systems. Indeed, in the absence of robust security solutions, malfunctions in a SoS may outweigh many of its benefits. This is due to the fact that the optimality of the SoS may easily fail if any of its subsystems is prone to dissent from the others. For example, they are vulnerable to a hidden attack when one subsystem gets hacked completely and tries to destabilize the entire system [3]–[6]. From a systematic point of view, each of the subsystem in an SoS can be considered as an agent in the presence of an adversarial agent (the hacked subsystem), who now is making decisions independently. The interaction between these subsystems can be modeled utilizing game theoretical approaches. Game theory is the study of decision-making problems in which there are multiple decision-makers and the optimality of their decisions depend on choices made by other decision makers. Game theory has been studied extensively as a modeling paradigm in the mathematical, social and economic sciences with a strong connection to multi-agent systems with multiple interacting decision-making entities [7]–[9]. One can nicely fit the modeling and simulation of a complex SoS to a game theoretical setting in order to mathematically model the interrelations of the adopted strategies by the subsystems. For example, one subsystem may decide to separate from the others and proceed in a direction that may be contrary or harmful to the entire SoS. In the context of Pursuit-Evasion problem, which can be viewed as a special case of SoS, three different strategies for the pursuers and evader have been studied in [10], [11]. This opens up an avenue of various possibilities for the remaining subsystems on how to cope with this new adversary.

In this paper, we will analyze dynamical SoSs in which due to a cyber attack one subsystem become adversarial in its decision making process with respect to the other subsystems. Without loss of generality and for simplicity of notation, we will consider only linear quadratic dynamical systems. This class of systems is not only important because of its mathematical tractability but also because of the insight that it provided once a solution is determined. The state dynamics of the entire system is modeled with linear differential equations, and the decision-making process is based on minimizing quadratic objective functions. We will assume that the conforming subsystems have similar objectives with a common goal of driving the system to a desired state while the dissenting subsystem is attempting to do the exact opposite, i.e drive the system away from that desired state. Clearly, the outcome of this conflict between one subsystem and the group of remaining subsystems will depend on the decision-making process of all subsystems. Intuitively, the most likely approach for the conforming subsystems is to group as a team and follow a unified team-based decision-making process. The team collective objective function may be

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modeled as a convex combination of the individual objective functions with a percent weight assigned to the contribution of each subsystem. This approach yields a 2-player game with the team of conforming subsystems as one decision-maker and the dissenting subsystem as the other. Within this framework several possible behavioral models of the conforming subsystems can be analyzed. For example, by taking the extreme cases of assigning the entire team effort to one subsystem, it is possible to assess the effectiveness of the one-on-one approach which, to some extent, represents a situation where the subsystems assess the effectiveness of each acting alone and making decisions on behalf of the entire team. The decision-making process for all of these scenarios will be developed by utilizing a game theoretic approach which will be based on the concept of Nash equilibrium. We are interested in the development of solutions that model cooperation among the conforming subsystems and produce fair and acceptable non-inferior (Pareto-optimal) strategies for the conforming subsystems that at the same time enjoy a Nash equilibrium property with the dissenting subsystem.

The concept of Noninferior Nash Strategy (NNS) between two competing teams of cooperating team members was first introduced by *Liu and Simaan* in the context of static systems [12]. In this paper, we extend this concept to a class of dynamical SoSs and study the properties of these strategies from a systematic point of view and compare them to the conventional Nash Strategy (NS) in [13], [14].

The remainder of the paper is organized as follows. The problem formulation is presented in section II. The two different possible decision-making strategies for the subsystems are analyzed and assessed in sections III and IV, respectively. An illustrative example is studied in section V. Concluding remarks are presented in section VI.

II. Problem Formulation

Without loss of generality and for mathematical tractability and simplicity of notation, we will consider only linear quadratic dynamical SoSs. Consider an SoS consisting of N+1 subsystems and whose state evolution is described by

$$\dot{x} = A(t)x + \sum_{i=1}^{N+1} B_i(t)u_i, \quad x(t_0) = x_0 \tag{1}$$

where $t \in [t_0, t_f]$ and $x \in \mathbb{R}^m$ is the combined state vector of all subsystems. Matrix $A \in \mathbb{R}^{m\times m}$ provides a description of the system dynamics and matrices B_i for $i = 1, 2, \ldots, N+1$ are corresponding decision matrices. The vectors u_i for $i = 1, 2, \ldots, N+1$ are the feedback-type decision variables for the subsystems which minimizes the following objective functions

$$J_i = \frac{1}{2}x^\intercal(t_f)S_{if}x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\big[x^\intercal Q_i(t)x + \sum_{j=1}^{N+1} u_j^\intercal R_{ij}(t)u_j\big]dt \tag{2}$$

for $i = 1, 2, \ldots, N+1$. In (2) symmetric weight matrices S_{if}, Q_i are positive semi-definite, and R_{ii} are positive definite. In the above description, each decision-maker is minimizing the objective function of its own subsystem, which is dependent on the decision variables of all the other subsystems as well as the state variable of entire system.

Now suppose that Subsystem N+1 is attacked in such a way that the attacker is able to infiltrate its database and change the sign of matrices from being positive definite to becoming negative definite. This subsystem now becomes dissenting. The new system dynamics now becomes

$$\dot{x} = A(t)x + \sum_{i=1}^{N} B_i(t)u_i + B_d(t)u_d, \quad x(t_0) = x_0 \tag{3}$$

where u_d is now the decision variable of the dissenting subsystem which is designed to minimize the objective function

$$J_d = \frac{1}{2}x^\intercal(t_f)S_{df}x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\big[x^\intercal Q_d(t)x + u_d^\intercal R_d(t)u_d + \sum_{j=1}^{N} u_j^\intercal R_{dj}(t)u_j\big]dt \tag{4}$$

where symmetric weight matrices $S_{df} < 0$, $Q_d < 0$ are negative definite and $R_d > 0$ is positive definite and $R_{dj} < 0$ are negative definite. The subscript d is used to characterize the dissenting subsystem instead of $N + 1$. The remaining conforming subsystems will minimize the following objective functions

$$J_i = \frac{1}{2}x^\intercal(t_f)S_{if}x(t_f) + \frac{1}{2}\int_{t_0}^{t_f}\big[x^\intercal Q_i(t)x + u_d^\intercal R_{id}(t)u_d + \sum_{j=1}^{N} u_j^\intercal R_{ij}(t)u_j\big]dt \tag{5}$$

for $i = 1, 2, \ldots, N$, where symmetric weight matrices $S_{if} \geq 0$, $Q_i \geq 0$ are positive semi-definite, and $R_{ij} > 0$ are positive definite and $R_{id} > 0$ are negative definite.

The conflict between the dissenting subsystem and the remaining subsystems is represented in the model (3)-(5), by the fact that S_{df} and Q_d are negative definite, meaning that decision-maker for dissenting subsystem is actually maximizing the state variables with minimum effort (as $R_d > 0$). The quadratic nature of the objective functions in this case might imply that while the conforming subsystems are trying to drive the sate of the entire system to an equilibrium position (the origin in this case), the dissenting subsystem is trying to prevent it from doing so.

III. Decision-Making Strategies

The subsystems in this system of systems can be viewed as players in a game where each must decide on a strategy to minimize its own objective function while taking into consideration the decisions of all other subsystems. In this paper, we consider two strategies; (a) the Noninferior Nash Strategy (NNS), and (b) the Nash Strategy (NS).

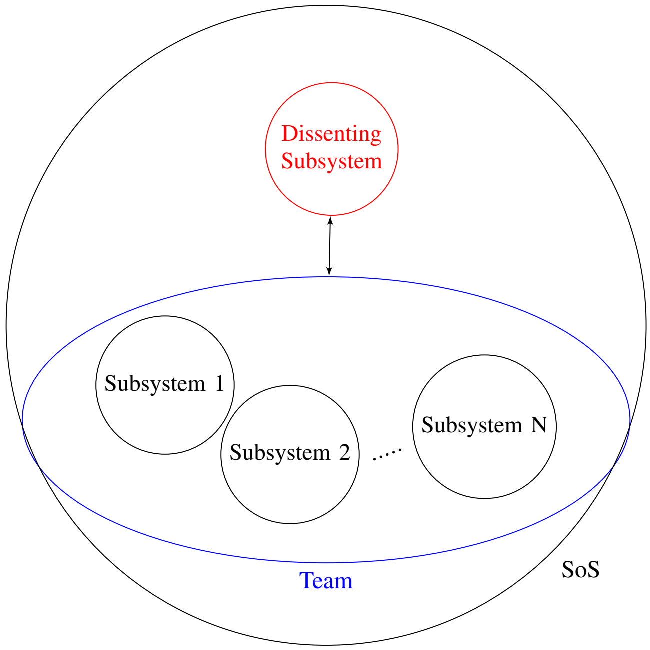

Fig. 1. Noninferior Nash Strategies for an SoS with a dissenting subsystem (denoted by red) and conforming subsystems (denoted by Subsystem 1 through N and blue).

A. Noninferior Nash Strategies (NNS)

The dissenting subsystem may cause the remaining subsystems to group as a team of cooperating members resulting in a game between two systems; the team of subsystems on one hand and the dissenting subsystem acting by itself on the other hand, as illustrated in Figure 1. The cooperation among the conforming subsystems is formulated as a convex combination of their objective functions. That is, the team objective function becomes

$$J_T = \sum_{i=1}^{N} \alpha_i J_i \tag{6}$$

where

$$\sum_{i=1}^{N} \alpha_i = 1, \quad \alpha_i \geq 0 \tag{7}$$

and J_i for $i = 1, 2, ..., N$ as they are defined in (5). The weight α_i can be viewed as the relative contribution of Subsystem i to the team.

One should notice that for the NNS, the weights α_i are nonnegative scalars for the conforming subsystems in the team. This requirement, in the extreme case allows for the assignment of the entire team effort to one subsystem. Such an assignment will make it possible to assess the effectiveness of one-on-one approach which represent a situation where one subsystem making decisions on behave of the entire team. So the objective function for team of conforming subsystems becomes

$$J_T = \frac{1}{2} x^\intercal S_{T_f} x \Big|_{t=t_f} + \frac{1}{2} \int_{t_0}^{t_f} \left[x^\intercal Q_T x + u_d^\intercal R_{Td} u_d + u_T^\intercal R_T u_T \right] dt \tag{8}$$

where

$$S_{T_f} = \sum_{i=1}^{N} \alpha_i S_{if}, \tag{9a}$$

$$Q_T = \sum_{i=1}^{N} \alpha_i Q_i, \tag{9b}$$

$$R_{Td} = \sum_{i=1}^{N} \alpha_i R_{id}, \tag{9c}$$

$$R_{Tj} = \sum_{i=1}^{N} \alpha_i R_{ij}, \quad j = 1, 2, \ldots, N, \tag{9d}$$

$$R_T = blkdiag\big(R_{T1}, R_{T2}, \ldots, R_{TN}\big), \tag{9e}$$

and the objective function for dissenting subsystem can be rewritten as follows

$$J_d = \frac{1}{2} x^\intercal S_{df} x \Big|_{t=t_f} + \frac{1}{2} \int_{t_0}^{t_f} \left[x^\intercal Q_d x + u_d^\intercal R_d u_d + u_T^\intercal R_{dT} u_T \right] dt \tag{10}$$

where

$$R_{dT} = blkdiag\big(R_{d1}, R_{d2}, \ldots, R_{dN}\big). \tag{11}$$

Vector $u_T = [u_1^\intercal, u_2^\intercal, ..., u_N^\intercal]^\intercal$ is the combined control vector of the conforming subsystems as one team. The function $blkdiag(.)$ maps the set of input matrices to a larger block diagonal matrix with all zero off-block-diagonal elements. Also, the conforming subsystem's decision variables are combined as one vector but it will be assigned back to each subsystem later at the implementation stage. Consequently, the system dynamics of (3) can be reorganized as follows

$$\dot{x} = Ax + B_T u_T + B_d u_d \tag{12}$$

where $B_T = [B_1 \ B_2 \ \ldots \ B_N]$. The system dynamics (12) and objective functions (8) and (10) form a two-system Linear Quadratic game. The feedback NNS can be obtained as follows

$$u_d^* = -R_d^{-1} B_d^\intercal S_d x \tag{13a}$$

$$u_T^* = -R_T^{-1} B_T^\intercal S_T x \tag{13b}$$

where the matrices S_T and S_d are the solutions to the following coupled differential Riccati equations

$$\dot{S_T} + S_T \bar{A}_T + \bar{A}_T^\intercal S_T + Q_T + S_T B_T R_T^{-1} B_T^\intercal S_T + S_d B_d R_d^{-1} R_{Td} R_d^{-1} B_d^\intercal S_d = 0, \tag{14a}$$

$$\dot{S_d} + S_d \bar{A}_T + \bar{A}_T^\intercal S_d + Q_d + S_d B_d R_d^{-1} B_d^\intercal S_d + S_T B_T R_T^{-1} R_{dT} R_T^{-1} B_T^\intercal S_T = 0, \tag{14b}$$

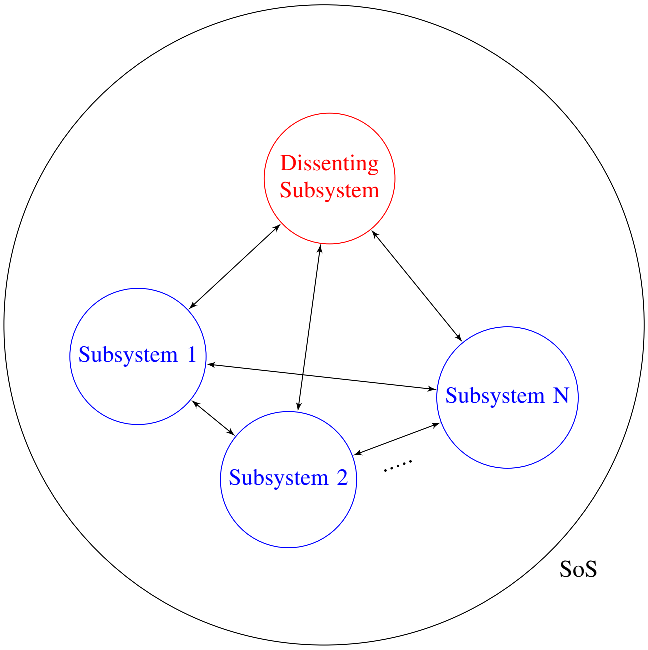

Fig. 2. Nash Strategies for an SoS with a dissenting subsystem (denoted by red) and conforming subsystems (denoted by Subsystem 1 through N and blue).

with boundary condition $S_T(t_f) = S_{T_f}$ and $S_d(t_f) = S_{df}$. The matrix $\bar{A}_T$ in (14) represents the system dynamics after implementation of the decision variables, which becomes

$$\bar{A}_T = A - B_T R_T^{-1} B_T^{\intercal} S_T - B_d R_d^{-1} B_d^{\intercal} S_d. \tag{15}$$

Note that by assigning different values of α_i, different teams with different strengths and weaknesses can be formed. Clearly, the opposing player is faced with a team with different possible characteristics. The weight α_i can be interpreted as the fraction of the total effort that each subsystem is contributing towards the team.

Remark. It is important to mention that, we had combined the decision variables of the conforming subsystems in order to derive the NNS. Now that we have determined u_T as in (13b), the team strategy can be reallocated to the corresponding subsystems. As a result, the conforming subsystems' decision variables will be

$$u_i^* = -R_{Ti}^{-1} B_i^{\intercal} S_T x, \quad i = 1, 2, \dots, N. \tag{16}$$

and the dissenting subsystem's decision variable will be

$$u_d^* = -R_d^{-1} B_d^{\intercal} S_d x. \tag{17}$$

B. Nash Strategies (NS)

The feedback NS for this game will take place if the dissenting subsystem causes the conforming subsystem to disintegrate and act as independent decision-making subsystems. The NS can be derived in the standard form as follows [13], [14]

$$u_d^* = -R_d^{-1} B_d^{\intercal} S_d x, \tag{18a}$$
$$u_i^* = -R_{ii}^{-1} B_i^{\intercal} S_i x, \quad i = 1, 2, ..., N, \tag{18b}$$

where the matrices S_d and S_i for all $i = 1, 2, ..., N$ satisfy the following (N+1)-coupled Differential Riccati Equations (DRE),

$$\dot{S}_d + S_d \bar{A} + \bar{A}^{\intercal} S_d + Q_d + S_d B_d R_d^{-1} B_d^{\intercal} S_d + \sum_{j=1}^{N} S_j B_j R_{jj}^{-1} R_{dj} R_{jj}^{-1} B_j^{\intercal} S_j = 0, \tag{19a}$$

$$\dot{S}_i + S_i \bar{A} + \bar{A}^{\intercal} S_i + Q_i + S_d B_d R_d^{-1} R_{id} R_d^{-1} B_d^{\intercal} S_d + \sum_{j=1}^{N} S_j B_j R_{jj}^{-1} R_{ij} R_{jj}^{-1} B_j^{\intercal} S_j = 0, \tag{19b}$$

with boundary conditions $S_d(t_f) = S_{df}$ and $S_i(t_f) = S_{if}$. The matrix $\bar{A}$ in (19) represents the system dynamics after implementation of the decision variables, which becomes

$$\bar{A} = A - \sum_{j=1}^{N} B_j R_{jj}^{-1} B_j^{\intercal} S_j - B_d R_d^{-1} B_d^{\intercal} S_d. \tag{20}$$

IV. Performance Assessment

For each of the strategies developed in previous section, the performance of each subsystem can be assessed by determining the value of the corresponding objective function.

A. Assessment of the Noninferior Nash Strategies

With the NNS decision strategies determined as in (16) and (17), then the state dynamics of the system would be

$$\dot{x} = \bar{A}_T x \tag{21}$$

where $\bar{A}_T$ is defined in (15). Therefore, the values of objective functions can be determined as

$$J_d^{NNS} = \frac{1}{2} x^{\intercal}(t_0) P_d(t_0) x(t_0) \tag{22a}$$
$$J_i^{NNS} = \frac{1}{2} x^{\intercal}(t_0) P_i(t_0) x(t_0), \quad i = 1, 2, \dots, N \tag{22b}$$

where $P_d(t)$ satisfies the following linear ordinary differential equation which depends on the solutions of (14) for S_d and S_T:

$$\dot{P}_d + P_d \bar{A}_T + \bar{A}_T^{\intercal} P_d + Q_d + S_d B_d R_d^{-1} B_d^{\intercal} S_d + \sum_{j=1}^{N} S_T B_j R_{Tj}^{-1} R_{dj} R_{Tj}^{-1} B_j^{\intercal} S_T = 0, \tag{23}$$

with boundary condition $P_d(t_f) = S_{df}$. Similarly, $P_i(t)$ satisfies the following linear ordinary differential equation

$$\dot{P}_i + P_i \bar{A}_T + \bar{A}_T^{\intercal} P_i + Q_i + S_d B_d R_d^{-1} R_{id} R_d^{-1} B_d^{\intercal} S_d + \sum_{j=1}^{N} S_T B_j R_{Tj}^{-1} R_{ij} R_{Tj}^{-1} B_j^{\intercal} S_T = 0, \tag{24}$$

with boundary condition $P_i(t_f) = S_{if}$ and for all $i = 1, 2, \dots, N$.

B. Assessment of the Nash Strategies

With the NS decision strategies determined as in (18a) and (18b), then the state dynamics of the system would be

$$\dot{x} = \bar{A}x \tag{25}$$

where $\bar{A}$ is defined in (20). Therefore, the values of objective function for each subsystem can be evaluated

$$J_d^{NS} = \frac{1}{2}x^\intercal(t_0)P_d(t_0)x(t_0) \tag{26a}$$

$$J_i^{NS} = \frac{1}{2}x^\intercal(t_0)P_i(t_0)x(t_0), \quad i = 1, 2, \ldots, N \tag{26b}$$

where $P_d(t)$ satisfies the following linear ordinary differential equation which depends on the solutions of (19) for S_d and S_i:

$$\dot{P}_d + P_d\bar{A} + \bar{A}^\intercal P_d + Q_d + S_d B_d R_d^{-1} B_d^\intercal S_d + \sum_{j=1}^{N} S_j B_j^\intercal R_{jj}^{-1} R_{dj} R_{jj}^{-1} B_j^\intercal S_j = 0, \tag{27}$$

with boundary condition $P_d(t_f) = S_{df}$. Similarly, $P_i(t)$ satisfies the following

$$\dot{P}_i + P_i\bar{A} + \bar{A}^\intercal P_i + Q_i + S_d B_d R_d^{-1} R_{id} R_d^{-1} B_d^\intercal S_d + \sum_{j=1}^{N} S_j B_j R_{jj}^{-1} R_{ij} R_{jj}^{-1} B_j^\intercal S_j = 0, \tag{28}$$

with boundary condition $P_i(t_f) = S_{if}$ and for all $i = 1, 2, \ldots, N$.

V. Simulation

In this section, we consider an example of an SoS consisting of three subsystems in which the third subsystem has been hacked and becomes dissenting while the other two remain conforming. Each subsystem is characterized by a two-dimensional state vector and a two-dimensional decision vector. The dynamic behavior of the entire system can be modeled by the following six-dimensional differential equation

$$\dot{x} = Ax + B_1u_1 + B_2u_2 + B_du_d \tag{29}$$

where $x = \begin{bmatrix} z_{1S1} & z_{2S1} & z_{1S2} & z_{2S2} & z_{1d} & z_{2d} \end{bmatrix}^\intercal \in \mathbb{R}^6$ is the combined state vector of entire system and z_{jSi} is the j-th state of Subsystem i and z_{jd} is the j-th state of dissenting subsystem. The system dynamics matrix $A = -blkdiag\{\mathbf{1}_{2\times2}, \mathbf{1}_{2\times2}, \mathbf{1}_{2\times2}\}$ is block diagonal, where $\mathbf{1}_{2\times2}$ is a matrix with all entries equal to one. Also, matrices $B_1 = [\mathbf{e}_3 \; \mathbf{e}_4]$, $B_2 = [\mathbf{e}_5 \; \mathbf{e}_6]$ and $B_d = [\mathbf{e}_1 \; \mathbf{e}_2]$ where $\mathbf{e}_i \in \mathbb{R}^6$ is the standard basis vector in direction i. This means that the subsystems have internally coupled state dynamics and each can decide to choose a decision variable of dimension 2 that can derive the corresponding internal state variables. The weight matrices of their objective functions (4) and (5) are as follows: $R_d = 10I_2$, $R_{11} = R_{22} = 10I_2$, $R_{id} = R_{di} = -I_2$ and $R_{ij} = R_{ji} = I_2$, $i \neq j$ for $i, j = 1, 2$ where I_2 is the identity matrix of dimension 2.

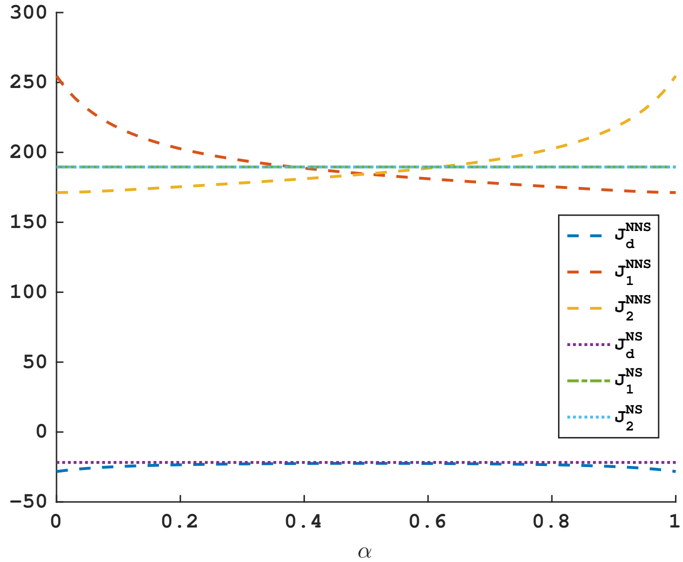

Fig. 3. Scenario 1- Costs of each subsystem versus the choice of α and for NNS comparing to NS.

Two scenarios are considered with different weight matrices as shown in Table I. The initial state vector is $x_0 = \begin{bmatrix} 5 & 5 & 5 & 0 & 0 & 5 \end{bmatrix}^\intercal$, and the simulation time is assumed to be $t \in [0, 2]$.

TABLE I

	S_{df}	Q_d	S_{1f}	Q_1	S_{2f}	Q_2
Scenario 1	$-1I_6$	$-1I_6$	$1I_6$	$1I_6$	$1I_6$	$1I_6$
Scenario 2	$-1I_6$	$-1I_6$	$20I_6$	$20I_6$	$30I_6$	$30I_6$

Following the definition of the NNS in section III-A, the contribution weights $\alpha_1 = \alpha$ and $\alpha_2 = 1 - \alpha$ correspond to the conforming subsystems 1 and 2, where $0 \leq \alpha \leq 1$. The objective function of each subsystem is calculated versus all possible values of α as an independent parameter.

In Scenario 1, as shown in Figure 3, one can conclude that the choice of $\alpha = 0.5$ corresponds to a fair outcome for both conforming subsystems. This yields $\alpha_1 = \alpha_2 = 0.5$, meaning equal fraction of effort by each conforming subsystem. It is due to the fact that in Scenario 1 both conforming subsystems have the same capability in terms of their objective functions and this choice of α results in equal contribution to the final outcome $J_1^{NNS} = J_2^{NNS} = 184.5$. As it can be seen in Figure 3, this fair choice correspond to a NNS that results in a better cost than the NS for both subsystems ($J_1^{NS} = J_2^{NS} = 189.6$).

In Scenario 2 and as shown in Figure 4, the fair choice of α that corresponds to equal costs for both conforming subsystems is $\alpha = 0.072$, (i.e. $\alpha_1 = 0.072$ and $\alpha_2 = 0.928$) which means the fraction of effort for conforming Subsystem 2 should be much higher than conforming Subsystem 1 to have a fair choice. This choice of α results in equal cost values of $J_1^{NNS} = J_2^{NNS} = 399$. However, as it can be seen in Figure 4, this fair choice correspond to a NNS that results in

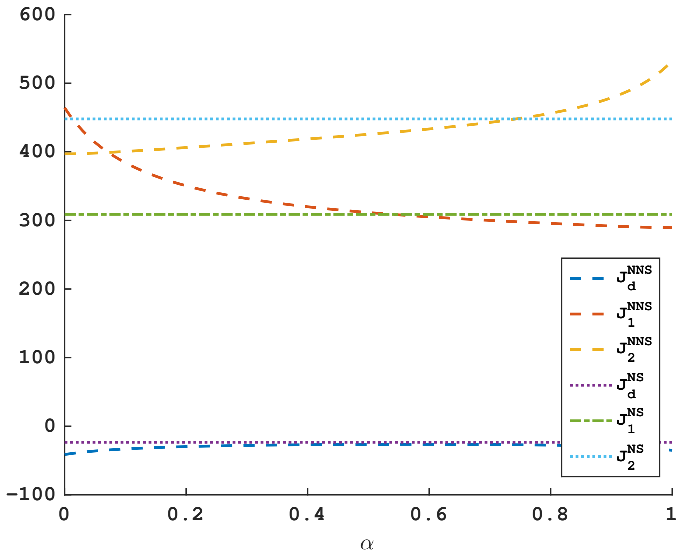

Fig. 4. Scenario 2- Costs of each subsystem versus the choice of α and for NNS comparing to NS.

a better cost than the NS for subsystem 2 ($J_2^{NS} = 447.9$), but a worse cost than the NS for subsystem 1 ($J_2^{NS} = 308.9$). So in this scenario, Subsystem 1 might not agree to cooperate with Subsystem 2 and instead chooses to separate and act on its own which results into disintegration of the entire SoS and all subsystems implementing the Nash Strategies.

In both cases, as can be seen in Figures 3 and 4, there is not much difference in the cost for the dissenting subsystem for different choices of α. In the Scenario 1, the cost for dissenting subsystem is $J_d = -34.77$ and in Scenario 2, it is $J_d = -25$. For the conforming subsystems, however, it can be seen in Figures 3 and 4, that there exists a range of α for which both subsystems will do better in NNS than NS options.

VI. Conclusion

In this paper, we considered the general problem of decision-making in Systems of Systems where one subsystem has been attacked and affected in such a way that it has become dissenting with the goals of the other conforming subsystems. We presented a theoretical analysis based on Game Theory for modeling the decision-making process in such Systems of Systems. We considered two Nash strategies as possible decision-making options for the subsystems. The first considers the conforming subsystems forming a team and acting with a unified strategy in opposition to the dissenting subsystem. The second considers each conforming subsystem acting independently on its own and developing its own Nash strategy in opposition to all other subsystems, which would happen when these subsystem could not agree on a specific team formation. These two solutions were derived in detail for the case of linear dynamics and quadratic objective functions. The Noninferior Nash Strategy for the team of conforming subsystems is formed with different weights representing the contribution of each member towards a common team objective function and enjoys a Nash equilibrium with the dissenting subsystem. We showed that the outcome of the decision-making process is heavily dependent on the degree of contribution of each member in the team. We have included an example that illustrates the results and demonstrates that there is a range of weights for which all conforming subsystems will do better belonging to a team than acting as independent decision-makers.

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