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SIMPE: A Simulation Platform for Multi-Player Pursuit-Evasion Problems

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***Abstract*—The pursuit-evasion problem has been studied extensively in the literature for decades. However, there are currently very limited interactive platforms that can assist a user in simulating different scenarios that may have practical relevance in certain applications. This paper presents SIMPE, an interactive simulation platform for simulating Pursuit-Evasion problems with many pursuers and one evader. This platform allows the user to interactively explore various scenarios of these types of problems by varying not only the parameters of the game but also the strategies that are available to the pursuers and evader. The exploratory nature of SIMPE allows the user to quickly and interactively determine the outcome of a particular complex scenario in a short period of time. Three different strategies that represent cooperating and/or not cooperating pursuers are available as options. In addition to various graphs and performance measures, the platform produces a visual display of the movement of the pursuers and evader which will terminate either by the evader escaping or being captured.**

I. Introduction

The multi-player Pursuit-Evasion (PE) refers to a game in which a number of pursuers try to capture one evader who is trying to escape. The solution of these types of games involves the development of movement strategies for all pursuers and evader as players in a game that will conclude with the evader either escaping or being captured. Following the work of *Berge* in 1957 [1] and *Isaacs* in 1965 [2] a variety of different formulations of PE games have been studied in the literature [3]–[9] including zero-sum [5], non-zero sum [10] and even Stackelberg [8], [9] games. The dynamics of the game can be formulated either in terms of velocity control [5], [11], or in terms of constant velocities but direction of movement control [12]–[14]. In recent years, multi-pursuers one-evader games have received considerable attention in the literature [3], [7], [15]–[17].

In [18], [19], three different strategies for N pursuers and one evader are considered in a multi-player Pursuit-Evasion game. Each pursuer's objective function reflects a desire to minimize the distance between itself and the evader while the evader's objective function reflects a need to escape by maximizing the distance between itself and a weighted measure of the distances between itself and the pursuers.

In this paper, and before we investigate multi-pursuers one-evader games, we consider an (N+1)-player linear quadratic game where N players have objective functions that include non-negative definite matrices while the remaining player has a similar objective function that includes negative definite matrices. Essentially, these games describe a situation where one player is directly opposed to the remaining group of players in the game. A set of sufficient conditions exists which guarantee exponential stability of the closed-loop Nash solutions [19]. Next we formulate the N-pursuers one-evader game as a special case of this (N+1)-player game and investigate three different strategies for the pursuers. These strategies include Non-cooperating pursuers, Cooperating pursuers and Greedy pursuers. On the other hand, we investigate different strategies for the evader that yield to different Nash equilibria against each pursuers' strategies.

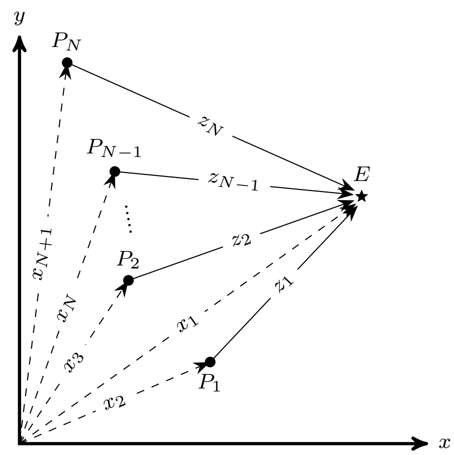

Fig. 1. Pursuers and the evader on an x-y coordinate system

In section III we present *SIMPE*, an agent-based simulation platform that supports a wide range of scenarios for the pursuers' and the evader's strategies. Its purpose is to help advance the understanding of pursuers strategies which include cooperating among themselves to capture the evader or acting on their own as independent players. The strength of *SIMPE* lies in its dynamic and interactive nature which makes it a user friendly platform where the user can change the parameters of the game and immediately sees the outcome of the resulting simulation. Additionally, *SIMPE* can be easily modified to simulate any multi-agent system in the presence of an adversary agent. We also present several illustrative examples that showcase the usefulness of this platform. Concluding remarks are presented in section IV.

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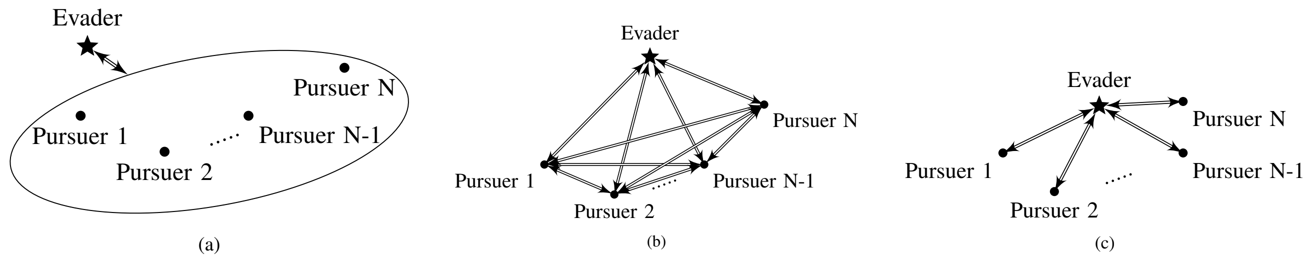


Fig. 2. Strategies of the PE game with the evader versus (a) Cooperative pursuers; (b) Non-cooperative pursuers; (c) Greedy pursuers.

II. BACKGROUND

Consider an M-player LQ Differential game with $M = N + 1$ where player 1 is the opposing player (Evader) and players 2 through M (Pursuers) are all against player 1 [19]. The game dynamics is linear in the standard form

$$\dot{z} = Az + B_1 u_1 + \sum_{j=2}^{M} B_j u_j, \quad z(t_0) = z_0; \tag{1}$$

where z is the state vector and u_1 through u_M are the player's control input vector. Matrix A is the open-loop system dynamics, matrices B_1 through B_M are player's input matrices, respectively. Let the objective functions of players be of the form

$$J_i = \frac{1}{2} z^\intercal(t_f) S_{if} z(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [z^\intercal(t) Q_i z(t) + u_i^\intercal(t) R_i u_i(t)]dt; \tag{2}$$

for $i = 1, 2, ..., M$, where matrices S_{1f} and Q_1 are symmetric negative definite matrices and R_1 is symmetric positive definite matrix. Also matrices S_{if} and Q_i are symmetric positive semi-definite matrices and R_i is symmetric positive definite matrix for $i = 2, 3, ..., M$. Basically, player 1 is trying to unregulate (drive the state vector of the system away from the origin) while players 2 through M are trying to regulate the system (drive the state vector of the system to the origin). The closed-loop Nash strategies for this game are of the form [10]

$$u_i^* = -R_i^{-1} B_i^\intercal S_i z \tag{3}$$

for $i = 1, 2, ..., M$, where S_i's satisfy the following M-coupled differential Riccati equations,

$$\dot{S}_i + S_i A + A^\intercal S_i + Q_i + S_i B_i R_i^{-1} B_i^\intercal S_i - \sum_{j=1}^{M} S_i B_j R_j^{-1} B_j^\intercal S_j + S_j B_j R_j^{-1} B_j^\intercal S_i = 0, \ S_i(t_f) = S_{if}. \tag{4}$$

A. Pursuit-Evasion Game Formulation

Following the game formulation in (1) and (2), the PE game with N pursuers and one evader can be formulated as a finite time LQ game in which the goal of the i-th pursuer is to minimize its distance with respect to the evader while the evader's goal is to maximize a conical combination of its distances from the pursuers.

Let $x_1 \in \mathbb{R}^n$ be the evader's position vector and $x_{i+1} \in \mathbb{R}^n$ be i-th pursuer's position vector and let

$$z_i = x_1 - x_{i+1}, \tag{5}$$

for $i = 1, 2, ..., N$, be the difference between position vector of the i-th pursuer and the evader as illustrated in Figure 1.

The system dynamics follows (1), where the state vector z is defined as $z = \begin{bmatrix} z_1^\intercal & z_2^\intercal & \dots & z_N^\intercal \end{bmatrix}^\intercal$ and $u_i \in \mathbb{R}^{n\times 1}$. The control input of each player is assumed to be velocity which leads to a system dynamics matrix of $A = \mathbf{0}$. The input matrix of the evader is $B_1 = \mathbf{1}_N \otimes I_n \in \mathbb{R}^{nN\times n}$ and the input matrices of the pursuers are $B_{i+1} = -\mathbf{e}_i \otimes I_n \in \mathbb{R}^{nN\times n}$ for $i = 1, 2, ..., N$. Matrix $I_n \in \mathbb{R}^{n\times n}$ is the identity, $\mathbf{e}_i \in \mathbb{R}^N$ denotes the vector with a 1 in the i-th coordinate and 0's elsewhere, $\mathbf{1}_N \in \mathbb{R}^{N\times 1}$ is a vector with all the entries equal to 1 and $\otimes$ is the Kronecker product. Dimension $n = 2$ which represents a PE game on the plane.

Each player should minimize the objective function as defined in (2), where the weights are chosen as follow

$$\begin{aligned} S_{1f} =& \ -diag\{f_e \mathbf{1}_N\} \otimes I_n, \\ Q_1 =& \ -diag\{q_e \mathbf{1}_N\} \otimes I_n, \\ R_1 =& \ r_e \otimes I_n, \\ S_{(i+1)f} =& \ diag\{f_{p_i} \mathbf{e}_i\} \otimes I_n, \\ Q_{i+1} =& \ diag\{q_{p_i} \mathbf{e}_i\} \otimes I_n, \\ R_{i+1} =& \ r_{p_i} \otimes I_n, \end{aligned} \tag{6}$$

where f_e, q_e, r_e and $f_{p_i}, q_{p_i}, r_{p_i}$ for $i = 1, 2, ..., N$ are positive scalar weights for the evader and i-th pursuer, respectively. Furthermore, we will assume that $t_0 = 0$.

In this paper, we consider three different strategies for the pursuers which have different Nash attributes as illustrated in Figure 2. In this section we are concerned with developing strategies for the pursuers. In doing so, the pursuers will assume that the evader will be using the corresponding Nash strategy but will not know whether the evader will implement that strategy or not. These strategies are discussed in the following subsections.

1) Cooperative Pursuers' Strategy : This game describes a situation where the pursuers act as one team against the evader as illustrated in Figure 2a. Essentially, it is a 2-player game where all the pursuers' controls are combined as one control vector. The cooperation among the pursuers can be formulated as a convex combination of their cost functions as $J_P = \sum_{i=1}^{N} \alpha_i J_{i+1}$, where $\sum_{i=1}^{N} \alpha_i = 1$ and $\alpha_i > 0$. The

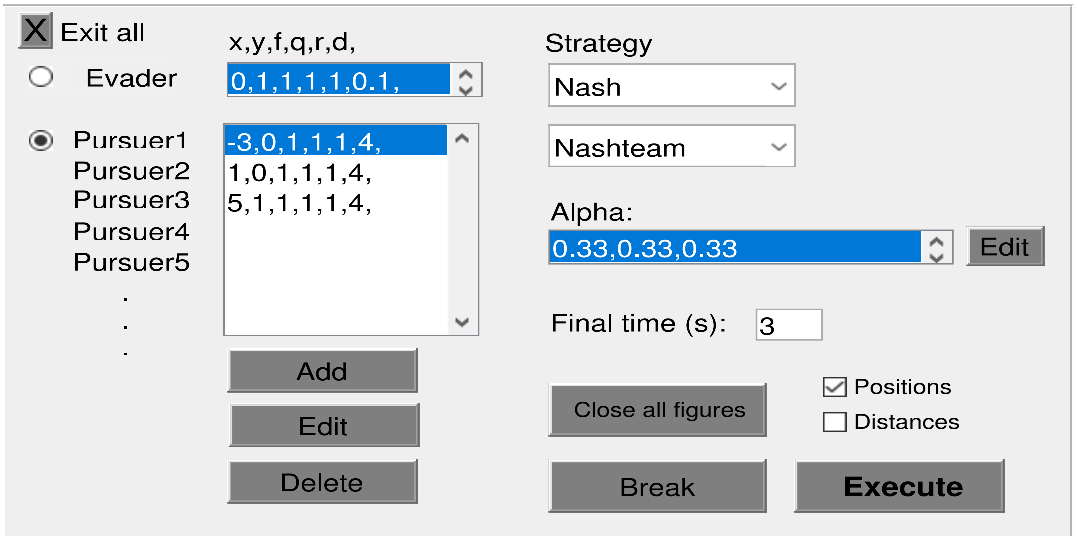
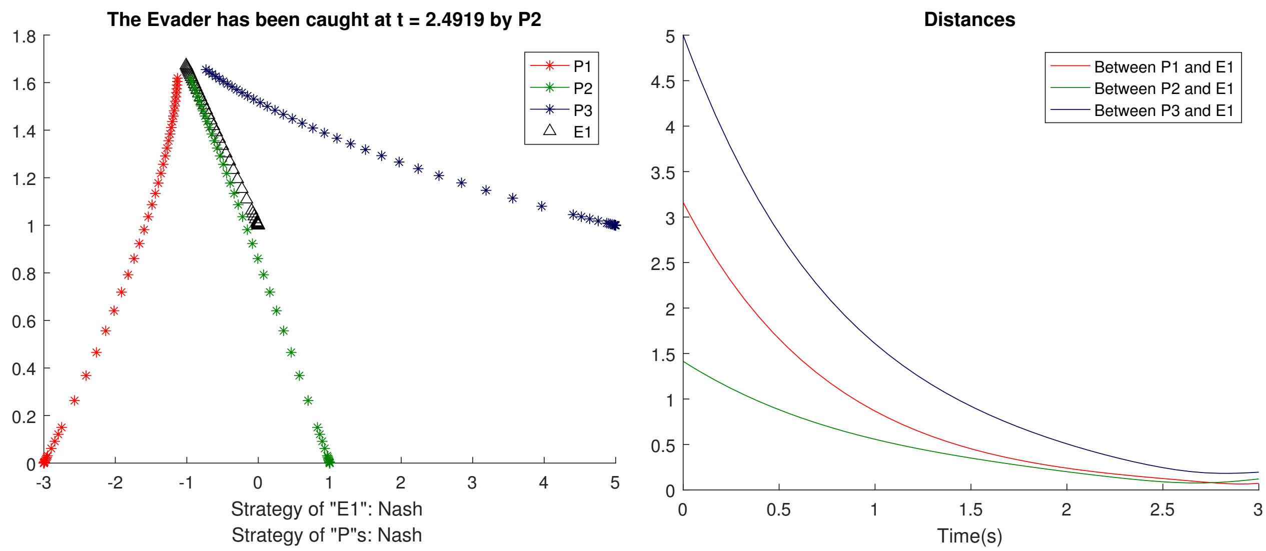


Fig. 3. *SIMPE*: the simulation platform



(a) (b)

Fig. 4. Output of *SIMPE* platform: (a) trajectories of players on x-y coordinate; (b) distances of pursuers to the evader at each instant of time.

weight α_i can be interpreted as the fraction of the total effort that the i-th pursuer is contributing towards the team.

2) Non-Cooperative Pursuers' Strategies: This game represent a situation where the pursuers are competing among themselves in their effort to capture the evader as illustrated in Figure 2b. The Nash equilibrium in this case will involve all pursuers and evader simultaneously, and the corresponding non-cooperative pursuers' strategies can be determined using (3) and (4).

3) Greedy Pursuers' Strategies: This game represent a situation where the pursuers are greedy. That is, each pursuer decides to ignore all other pursuers and pursue the evader on its own as illustrated in Figure 2c. Essentially, in this case we have N separate PE games between each pursuer and the evader. The pursuers' strategies are similar to the ones proposed in [1] where each pursuer moves exactly toward the evader at each instant of time, ignoring what the others are doing. For more details on the derivation of these three strategies refer to [19].

III. The Simulation Platform

A. SIMPE

SIMPE is a Matlab[1]-based simulation platform for Pursuit-Evasion games that assist the user with an interactive tool to explore a wide range of scenarios, strategies and game parameters. Its graphic user interface is shown in Figure 3. The platform inherits flexibilities that can support various type PE games, including arbitrary number of pursuers and their initial position on the x-y plane. It also includes different choice of strategies by each side of the game, and arbitrary choice of parameters for each individual player. It also allows the user

[1]The MathWorks Inc. Natrick, MA

to add/delete players and edit the game parameters. The user can choose to display the output either as the positions on the x-y coordinates or as the distances separating the pursuers from the evader as a functions of time.

The visual features of *SIMPE* help the user to better understand the impact of utilizing different strategies in specific situations and decide to implement the desired one in a real situation. It also allows the user to define different capturing criteria and to simulate the resulting trajectories and distances as a function of time. It can predict the outcome of the game as either the evader escaping or being captured after a specific termination time which can be chosen by user arbitrarily as can be seen in Figure 3. The output of *SIMPE* for each case would be one short movie illustrating the movement of players on the x-y coordinates and/or one figure showing distances of pursuers with respect to the evader at each instant of time as it is shown in Figure 4. The simulation can be terminated at any time by pressing the "Break" button.

In a practical PE game, however, full knowledge of the parameters and also the strategy used by the other side may not be available to the other side of the game. In this case, the pursuers might be interested in determining the probabilities of capturing the evader under some statistical information and for several possible strategies used by the evader. The core simulator for this case is done, however, it is not included to the current version of *SIMPE*.

B. Parameters of the Pursuers and Evader

As it can be seen in Figure 3, on the left side, an arbitrary number of pursuers can be added to *SIMPE* and also the parameters of the players can be arbitrarily chosen. In front of each player, there is a list consisting of six parameters that are entered in the following sequence. The first two represent the x and y coordinates of each player. The next three represent f, q and r parameters of the players' cost functions as defined in (6). As can be seen in Figure 3 and equation (6), parameters f and q should be non-negative real numbers and r should be a positive number for all players. Finally the last parameter, denoted by d, is the radius of capture which is a small positive number that represents a circle around the evader that terminates the game if at least one pursuer enters into it. Additionally for the cooperative strategy of pursuers, there is another list of parameters as α_i which can be chosen by the user to denote the fraction of effort by each pursuer towards capturing the evader as defined in Section II-A1.

C. Strategies for the Pursuers and Evader

SIMPE provides the user with different choices of strategies for each side of the game. On one hand as defined in Section II-A, pursuers have the following three strategies chosen from a drop-menu

1) Non-cooperative Nash denoted by *Nash*
2) Cooperative Nash denoted by *Nashteam*
3) Greedy denoted by *Greedy*

On the other hand, the evader can choose the corresponding Nash strategy to each of pursuers' strategies mentioned above. Additionally, as by the definition of Greedy strategy, the evader has to choose the pursuer that it decides to play the Greedy Nash strategy against. Finally, the evader can also choose to play Greedy Nash strategy versus the closest pursuer at each instant of time. So the evader has the following four types of strategies chosen from a drop-menu

1) Non-cooperative Nash denoted by *Nash*
2) Cooperative Nash denoted by *Nashteam*
3) Greedy Nash versus the pursuer k denoted by *Nash=k*
4) Greedy Nash versus the closest pursuer denoted by *Nashclosest*

In the following subsection a few illustrative example will showcase the applications of *SIMPE*.

D. Examples

Consider a four-pursuer one-evader game with parameters as described in Table I and Figure 5. The terminal time is assumed to be $t_f = 3$ and radius of capture is assumed to be $d = 0.1$ for the evader. In the cooperative pursuers game, $\alpha_i = 0.25$ are used in the team objective functions, representing equal contributions by all pursuers. The resulting trajectories for different strategies for pursuers and the evader are illustrated in Figures 6-8. Fig. 6 corresponds to the strategy in Fig 2a, Fig. 7 corresponds to the strategy in Fig. 2b and Fig. 8 corresponds to the strategy in Fig. 2c.

TABLE I
POSITIONS AND PARAMETERS OF PLAYERS IN THE EXAMPLE

	Position	Parameters[1]		
		f	q	r
Evader	$[0, 1]$	-1	-1	1
Pursuer 1	$[-3, 0]$	1	1	1
Pursuer 2	$[1, 0]$	1	1	1
Pursuer 3	$[4, 0.5]$	1	1	1
Pursuer 4	$[5, 1]$	1	1	1

1. Parameters are the variables of weight matrices in (6).

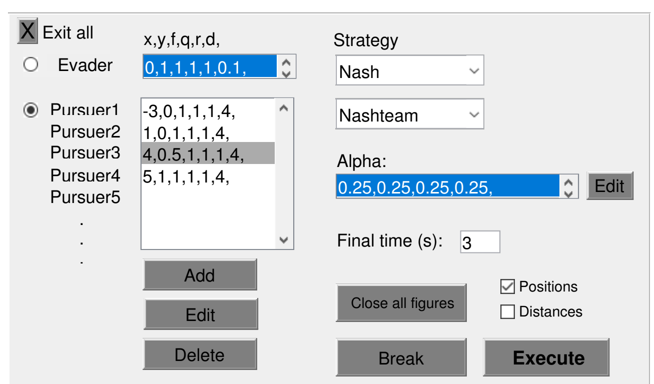
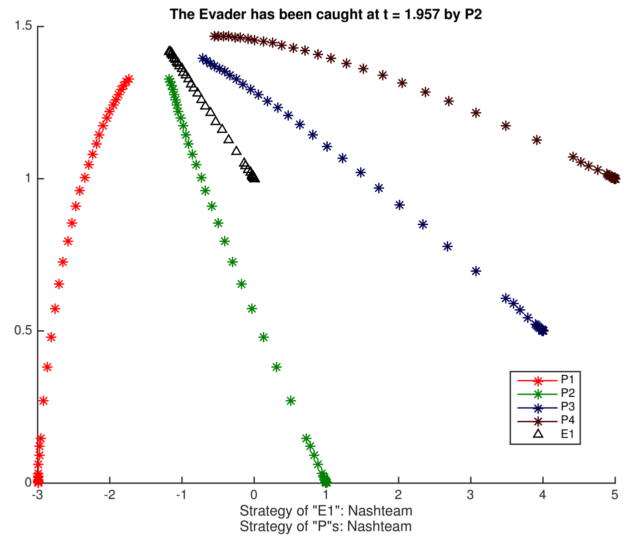


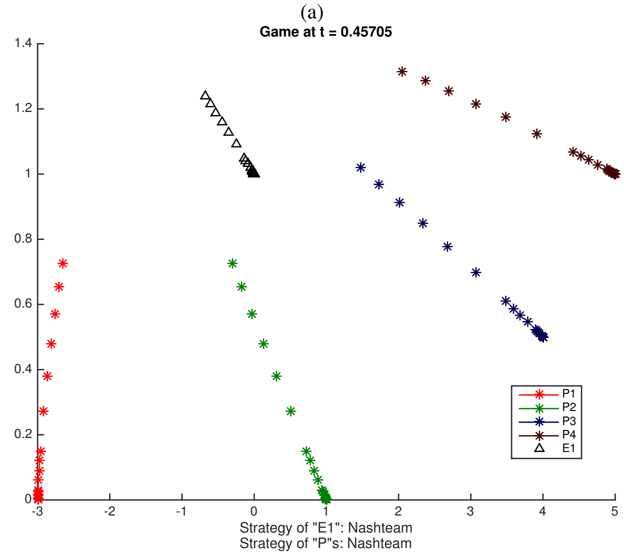
Fig. 5. Example simulated by *SIMPE*

IV. CONCLUSION

In this paper we have presented *SIMPE*, a Matlab-based platform for simulating Pursuit-Evasion games.This simulation platform can be adapted to simulate any multi-agent



(a)



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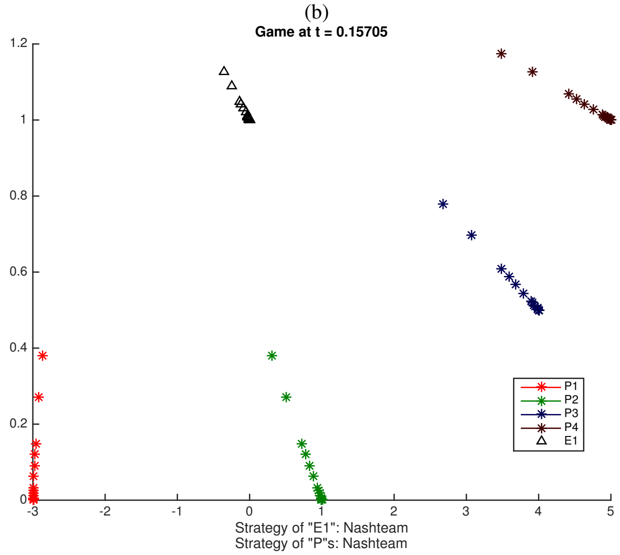
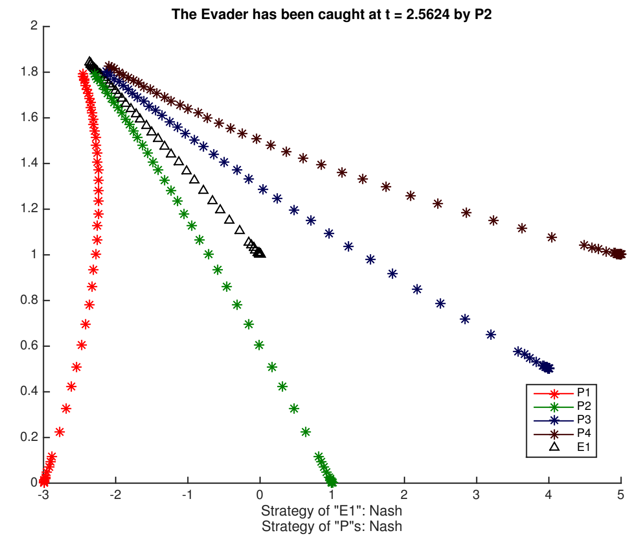
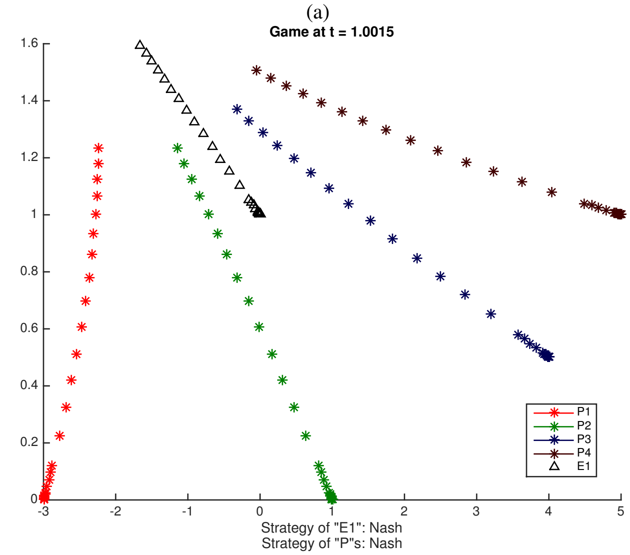


(c)

Fig. 6. Example: Output of SIMPE Shown at Different Times During the Evolution of the Game for Nashteam strategy.



(a)



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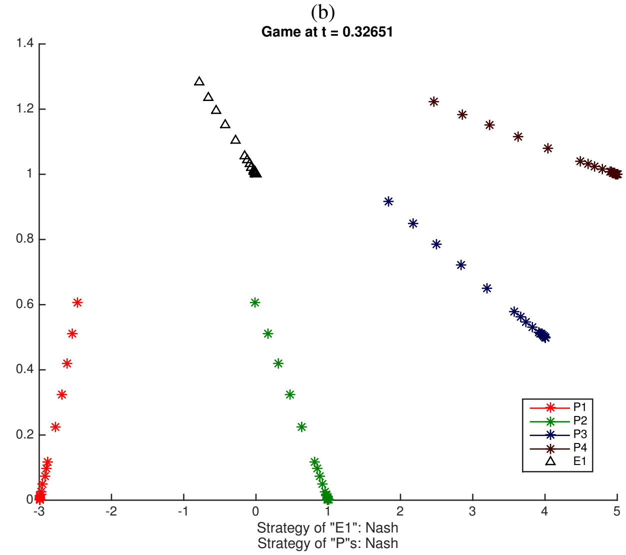

(c)

Fig. 7. Example: Output of SIMPE Shown at Different Times During the Evolution of the Game for Nash strategy.

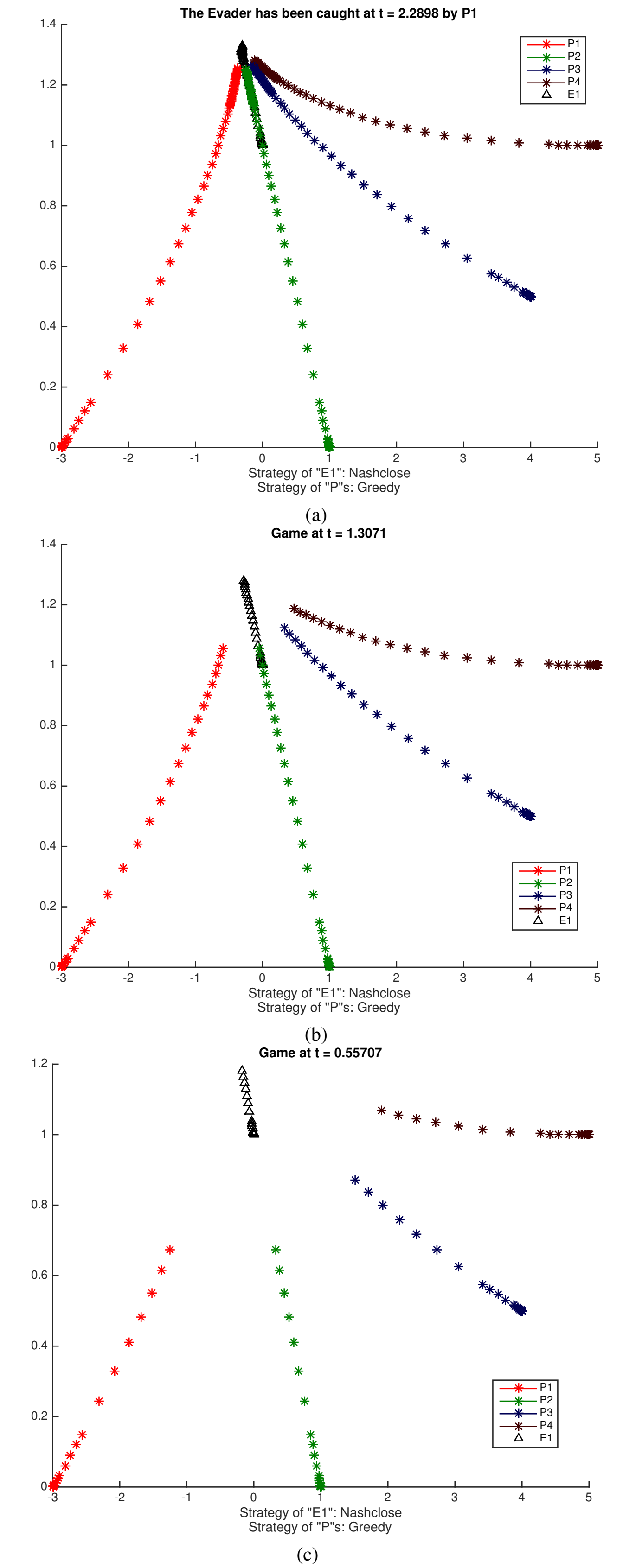

Fig. 8. Example: Output of SIMPE Shown at Different Times During the Evolution of the Game for Greedy strategy.

systems in presence of an adversary agent. *SIMPE* is used to study different pursuers strategies in a multi-pursuers one-evader PE game, which led to a deeper understanding about characteristics of these strategies. They include Cooperative, Non-cooperative and Greedy pursuers with the corresponding Nash strategies for the evader. Also the platform is designed to provide enough flexibility that can cover a wide range of variations of Pursuit-Evasion games. Several examples are used to illustrate the practical value of *SIMPE* and simulate several different scenarios that may be used by the pursuers to capture the evader.

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