Multi-Pursuer Pursuit-Evasion Games Under Parameters Uncertainty: A Monte Carlo Approach

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***Abstract*—The traditional pursuit-evasion game considers a situation where one or more pursuers try to catch an evader, while the evader is trying to escape. Instead of moving entities, a more general scope of the pursuit-evasion game could also consider systems of systems. In a system consisting of many subsystems, when one subsystem decides to dissent and operate in a manner that is inconsistent with the others, a game situation similar to the pursuit-evasion game occurs. The dissenting subsystem can be viewed as an evader and the remaining conforming subsystems can be viewed as pursuers who oppose and try to prevent the dissention from succeeding. This is a more general system of systems, approach to the pursuit-evasion problem where the players are now subsystems of a system rather than moving entities. Clearly, any control strategy by any one of the subsystems to separate needs to be assessed against a variety of options that the remaining subsystems may use. In this paper, we consider a game where the pursuer subsystems consider three possible strategies: (1) act independently as Nash players, (2) act optimally as a team, and (3) act individually as greedy pursuers. The evader subsystem, on the other hand, will consider strategies against all possible strategies by the pursuer subsystems. We assume that no subsystem knows which strategies the other subsystems are implementing and none of the subsystems has information about any of the parameters in the objective functions of the other subsystems. We deal with these uncertainties by first developing the Nash strategies for each of the resulting games for all possible options available to both sides. Given the prevailing parameter uncertainty in developing the strategies, we perform a Monte Carlo analysis to determine probabilities of success (or failure) for each of the strategies considered by each side. We illustrate the results using two simulation scenarios of a pursuit-evasion example consisting of three pursuers moving on a plane and chasing one evader.**

I. INTRODUCTION

Many large enterprises consist of a large number of interacting subsystems. These systems of systems (SoS) often operate optimally when all subsystems have a harmonious non-conflicting relationship among themselves. When one subsystem decides to operate in a manner that is not consistent with the others, the operation of the entire enterprise suffers resulting in an adversarial environment that affects not only the behavior of the dissenting subsystem but also possibly the behavior of the other conforming subsystems. The disruption caused by one dissenting subsystem may result in the entire system of systems disintegrating and behaving in a non-cooperative manner within itself. These types of complex systems of systems are treated best using concepts from game theory. If the systems are dynamic in nature, then differential game theory is the appropriate framework to do the analysis.

Traditional Pursuit-Evasion (PE) problems refer to a game in which one or more pursuers tries to capture one or more evaders who are trying to escape. Ever since the appearance of the book by *Rufus Isaacs* [1] these types of problems have been studied extensively in the differential game literature. Most of the problems considered involved one evader and one pursuer [2]–[10] and have been investigated from different aspects as zero-sum games [6], nonzero-sum games [11]–[13] and even as Stackelberg games [9], [10]. More recently, problems involving many pursuers and one evader have received considerable attention in the literature [2], [8], [14], [15]. In [15], a multi-pursuer one-evader game has been studied in the presence of limited information for the pursuers. The game is formulated as a linear-quadratic system in which the pursuers are cooperating together and play as a team against the evader. This approach yielded a final strategy for the pursuers that is independent of the systems initial state and satisfies a Nash equilibrium against the evaders strategy with respect to the best achievable performance indices. In [16], a game decomposition is introduced and applied to a two-pursuer one-evader game, which considerably reduces the time complexity of the solution. However, it has not considered whether the decomposition always is the best strategy for the pursuers in other situations. Among all of these, a few works have studied the effectiveness of different strategies in dynamic or static games, in the presence of game parameters uncertainty [17]–[20].

One of the most representative examples of the system of systems approach to the pursuit evasion game is a system of entities such as UAVs or robots moving together in a coordinated fashion to perform a common task. If one entity decides to separate from the group and operate on its own, the remaining entities will face the problem of deciding how to proceed. For example, one UAV or one robot may decide to leave the group and proceed on its own with an objective that may be contrary or harmful to the group's common objective. The remaining entities may or may not agree on how to react. They may decide to pursue a group strategy in an attempt to prevent it from accomplishing its objective or they may disagree on the proper course of action resulting in a system-wide breakdown and producing a non-cooperative environment where each entity acts unilaterally on its own. In many ways, these problems are very similar in formulation to the traditional multi-pursuer pursuit-evasion problems.

As mentioned above, in this paper we consider a more

This work was supported in part by the US National Science Foundation under grants ECCS-1308928 and CCF-0956501.

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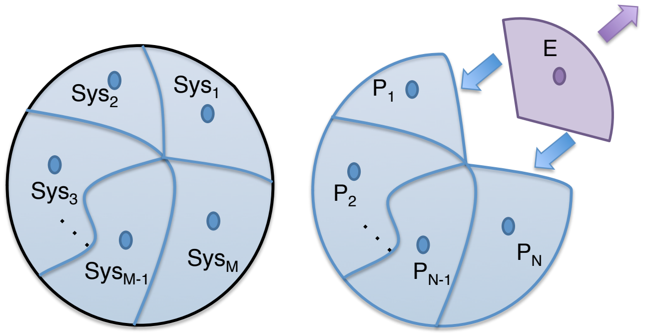


(a) (b)

Fig. 1. (a) System of Systems, (b) One subsystem decides to separate and the others try to prevent it from separating.

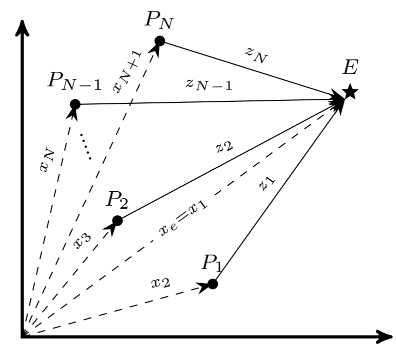


Fig. 2. State Vectors of Pursuers and Evader Subsystems

general SoS approach to the multi-pursuers one-evader problem where the pursuers and evader are now interconnected subsystems in a large enterprise. When one subsystem decides to dissent, it creates a situation similar to the pursuit-evasion game where the dissenting subsystem is the evader and the remaining subsystem are the pursuers trying to develop a strategy to bring the evader subsystem back into compliance (Figure 1). The goal of each subsystem may be expressed in terms of minimizing a representative cost function that depends on the action of all subsystems. Clearly, such a problem is filled with uncertainties and its solution is not straightforward. First, once the harmonious relationship among the subsystems is broken by the dissenting subsystem, it is not clear what strategy the conforming subsystems will implement as a response against the dissenting subsystem. Second, to be able to implement any strategy, all subsystems are faced with the uncertainty of not having knowledge of the various parameters in the cost functions of other subsystems. In this paper, we will deal with the first uncertainty by considering three possible strategies for the conforming subsystems: (1) act independently as non-cooperating Nash players, (2) act collectively as a cooperating team, and (3) act unilaterally as greedy players. We will deal with the second uncertainty by performing a Monte Carlo analysis over the entire parameter space of all cost functions to determine probabilities of success (or failures) by the conforming subsystems to accomplish their objective.

The remainder of the paper is organized as follows. The problem formulation is presented in section II. The three different possible game strategies for the conforming subsystems (i.e. pursuers) are analyzed and presented in section III. An illustrative pursuit-evasion example of moving entities involving three pursuers and one evader, and the corresponding Monte Carlo simulation results with two different scenarios are shown in section IV. Concluding remarks are discussed in section V.

II. Problem Formulation

In this paper, we consider a system of systems approach to the pursuit evasion problem. This approach considers a system consisting of many subsystems in which one subsystem decides to separate and operate in a manner that is inconsistent with the remaining subsystems. Using the same terminology as the standard pursuit evasion problem, and without loss of generality, we will refer to the dissenting subsystem as the evader and to the remaining conforming subsystems as the pursuers. All of the subsystems in the system are referred to as players in the game. Let $M = N+1$ be the total number of players and let N be the number of pursuers. Without loss of generality we will label the evader as player 1 and the pursuers as players 2, ..., M. To make the problem tractable, we will consider a linear quadratic system where the state variables representing each player is $x_i \in \mathbb{R}^n$ for $i = 1, 2, \dots, M$ and n can be any positive integer. For simplicity, it is assumed that the internal dynamics of each subsystem is as follows

$$\dot{x}_i = u_i \tag{1}$$

where $u_i \in \mathbb{R}^{n\times 1}$ is the i-th player's control input vector. Let's also define the generalized state vector as $z = \begin{bmatrix} z_1^\intercal & z_2^\intercal & \dots & z_N^\intercal \end{bmatrix}^\intercal$ where $z_i \in \mathbb{R}^n \quad \forall i \in \{1, ..., N\}$ is the displacement vector of the i-th pursuer with respect to the evader as follows

$$z_i = x_e - x_{i+1} \qquad \forall i \in \{1, 2, \dots, N\}; \tag{2}$$

where $x_e = x_1$. For simplicity, the relationships between the state vector of each pursuer and the evader are illustrated in Figure 2. The goal of i-th pursuer is to minimize its displacement vector z_i while the evader's goal is to maximize the generalized state vector at the same time. The state space representation of the system in term of generalized state vector is as follows

$$\dot{z} = \sum_{i=1}^{M} B_i u_i, \quad z(0) = z_0; \tag{3}$$

where $u_i \in \mathbb{R}^{n\times 1}$ is control input vector of player i, $B_i \in \mathbb{R}^{nN\times n}$ is input matrix of player i and z_0 is the generalized initial state vector at $t = 0$. Considering the player 1 as the evader, $B_1 = \mathbf{1}_N \otimes \mathbf{I}_n$ and $B_i = -\mathbf{e}_{i-1} \otimes \mathbf{I}_n \quad \forall i \in \{2, ..., M\}$, respectively. The notation $\mathbf{1}_N \in \mathbb{R}^{N\times 1}$ denotes a vector with all the entries equal to 1, $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix, $\mathbf{e}_i$ denotes the vector with a 1 in the i-th location and 0's elsewhere and $\otimes$ is the Kronecker product. We formulate the PE game as a finite time Linear Quadratic (LQ) Game when player i minimizes the following cost function

$$J_i = \frac{1}{2}z(t_f)^\intercal S_{if} z(t_f) + \frac{1}{2}\int_0^{t_f}[z(t)^\intercal Q_i z(t) + u_i(t)^\intercal R_i u_i(t)]dt \quad \forall i \in \{1,2,...,M\}; \quad (4)$$

where $S_{if} \in \mathbb{R}^{nN\times nN}, Q_i \in \mathbb{R}^{nN\times nN}$ and $R_i \in \mathbb{R}^{n\times n}$ are symmetric weight matrices. The assumptions on these weight parameter matrices are as follows:

1) For the evader (player 1): $S_{1f} = S_{e_f} \leq 0, Q_1 = Q_e \leq 0$ and $R_1 = R_e \geq 0$.
2) For the pursuers (players 2, ..., M): $S_{(i+1)f} = S_{p_if} \geq 0$, $Q_{(i+1)} = Q_{p_i} \geq 0$ and $R_{(i+1)} = R_{p_i} \geq 0$ for $i = 1,2,...,N$.

Remark. While all players are minimizing their cost functions, we should note that since the terminal cost S_{1f} and trajectory cost Q_1 for the evader are negative semi-definite matrices, the evader is acturally maximizing the generalized state vector in its cost function (which is no longer convex). For the evader the weights in its cost function represent a desire to distance itself as much as possible from all the pursuers. For each pursuer, on the other hand, the weights in its cost function represent a desire to bring the evader as close as possible to itself.

To make the problem tractable, and for simplicity of notation, the weight matrices are chosen to be diagonal. The weights for the evader are chosen as follows:

$$\begin{aligned} S_{e_f} &= -diag\{f_{e_1}, ..., f_{e_N}\} \otimes I_n, \\ Q_e &= -diag\{q_{e_1}, ..., q_{e_N}\} \otimes I_n, \\ R_e &= r_e \otimes I_n, \end{aligned} \quad (5)$$

for $i = 1,2,...,N$, where f_{e_i}, q_{e_i} and r_e are non-negative scalar weights of evader with respect to i-th pursuer. The weights for the pursuers are chosen as follows:

$$S_{p_i} = f_{p_i}\mathbf{e}_i \otimes I_n,\ Q_{p_i} = q_{p_i}\mathbf{e}_i \otimes I_n,\ R_{p_i} = r_{p_i} \otimes I_n; \quad (6)$$

for $i = 1,2,...,N$, where f_{p_i}, q_{p_i} and r_{p_i} are positive scalar weights of i-th pursuer.

III. Control Strategies

The subsystems in this system of systems are now players in a game where each must decide on a strategy to minimize its own cost function while taking into consideration the actions of all other players. As mentioned earlier, the evader's strategy is to separate and distance itself from the pursuers. This action by the evader will cause the pursuers to react in several possible ways. In this paper, we consider three possible such reactions as illustrated in Figure 3; (A) a system breakdown resulting in a non-cooperative strategy among all players, (B) a cooperative teaming of the pursuers against the evader, and (C) a disintegration of the pursuers into individual greedy pursuers. Each of these three strategies have a different Nash attribute [11]. Existence conditions of these Nash strategies (with $S_{1f} \leq 0$ and $Q_1 \leq 0$) are discussed in [20].

A. All Pursuers Non-cooperative Nash Strategies (All Nash)

The dissenting subsystem may cause a complete breakdown of the system resulting in all pursuers and the evader adopting a Nash strategy. Such a strategy represents a total non-cooperative environment among all players and yields an environment where each player becomes focused on safeguarding itself against possible cheating by other players (as illustrated in Figure 3a). It is well known in this case that the closed-loop Nash strategy for each player is given by:

$$u_i^* = -R_i^{-1}B_iS_iz \quad \forall i \in \{1,2,...,M\} \quad (7)$$

where the matrices S_i are the solutions to the following M-coupled differential Riccati equations,

$$\dot{S}_i + Q_i + S_iB_iR_i^{-1}B_i^\intercal S_i - \sum_{j=1}^{M}(S_iB_jR_j^{-1}B_j^\intercal S_j + S_jB_jR_j^{-1}B_j^\intercal S_i) = 0,$$
$$S_i(t_f) = S_{if} \quad \forall i \in \{1,2,...,M\}. \quad (8)$$

B. Team Pursuers Nash Strategy (Team Optimal)

The dissenting subsystem may cause the remaining subsystems to group as a team of cooperating players resulting in a two-player game between the pursuers team and the evader (as illustrated in Figure 3b). Considering the problem formulation in Section II, the cooperation among the pursuers yields a team cost function that consists of a convex combination of the individual pursuers cost functions as follows:

$$J_P = \sum_{i=1}^{N}\alpha_i J_{p_i}; \quad (9)$$

where $J_{p_i} = J_{i+1}$ as in (4) for $i = 1,2,...,N$, $\alpha_i > 0$ as weight for the i-th pursuer and $\sum_{i=1}^{N}\alpha_i = 1$. That is

$$J_P = \frac{1}{2}z(t_f)^\intercal S_{P_f} z(t_f) + \frac{1}{2}\int_0^{t_f}[z(t)^\intercal Q_P z(t) + u_i(t)^\intercal R_P u_i(t)]dt \quad (10)$$

where

$$S_{P_f} = \sum_{i=1}^{N}\alpha_i S_{p_if},\ Q_P = \sum_{i=1}^{N}\alpha_i Q_{p_i},\ R_P = \sum_{i=1}^{N}\alpha_i R_{p_i}. \quad (11)$$

And the evader's cost function remains $J_e = J_1$ as:

$$J_e = \frac{1}{2}z(t_f)^\intercal S_{e_f} z(t_f) + \frac{1}{2}\int_0^{t_f}[z(t)^\intercal Q_e z(t) + u_e(t)^\intercal R_e u_e(t)]dt. \quad (12)$$

Also the system dynamics become

$$\dot{z} = B_Pu_P + B_eu_e, \quad z(0) = z_0; \quad (13)$$

where $u_P = [u_2^\intercal, u_3^\intercal, ..., u_M^\intercal]^\intercal \in \mathbb{R}^{Nn\times 1}$ and $u_e = u_1 \in \mathbb{R}^{n\times 1}$ are the combined input vector of the pursuers as a team and the evader's control input vector, respectively. The combined input matrix of the pursuers as a team becomes $B_P = -\mathbf{I}_N \otimes \mathbf{I}_n$ and the input matrix of the evader remains the same as $B_e = B_1$.

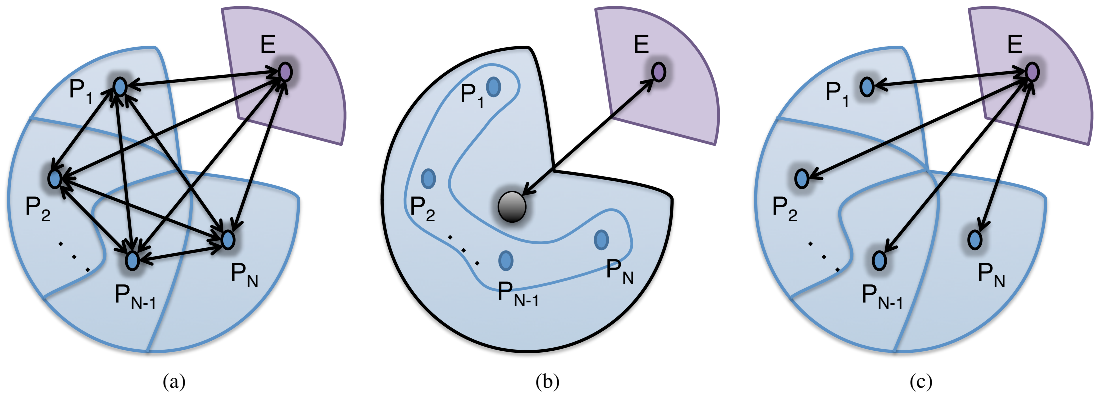


Fig. 3. Strategies of Conforming Subsystems (Pursuers), (a) Non-cooperating Pursuers All Nash Strategies (b) Team Optimal Pursuers Nash Strategy, (c) Greedy Pursuers Nash Strategies.

Equations (10), (12) and (13) form a finite time 2-player LQ Game in which the pursuers are cooperating as a team against the evader. To obtain the Pareto Front solution of cooperation among the pursuers, one can form J_P for different value of α_i which impacts the outcome of the game. Then the Team Optimal closed-loop Nash strategy can be obtained as follow

$$u_P^* = -R_P^{-1} B_P S_P z, \quad (14)$$
$$u_e^* = -R_e^{-1} B_e S_e z; \quad (15)$$

where S_P and S_e are the solutions to the following coupled differential Riccati equations

$$\dot{S_P} + Q_P - S_P B_P R_P^{-1} B_P^\intercal S_P - S_P B_e R_e^{-1} B_e^\intercal S_e - S_e B_e R_e^{-1} B_e^\intercal S_P = 0, \quad S_P(t_f) = S_{P_f}, \quad (16)$$

$$\dot{S}_e + Q_e - S_e B_e R_e^{-1} B_e^\intercal S_e - S_e B_P R_P^{-1} B_P^\intercal S_P - S_P B_P R_P^{-1} B_P^\intercal S_e = 0, \quad S_e(t_f) = S_{e_f}. \quad (17)$$

C. Greedy Pursuers Nash Strategies (Greedy)

The dissenting subsystem may cause each of the remaining subsystem to separate, each deciding to take the matter in its own hands and act individually against the evader irrespective of what the other pursuers decide to do (as illustrated in Figure 3c). Such a greedy strategy by the pursuers will result in N separate games and will be met by an evader strategy that considers the most favorable outcome for itself and ignores all the others. In this case, each game will be characterized by uncoupled players' dynamics as follows:

$$\dot{z}_i = B_{eg} u_{e_i g} + B_{p_i g} u_{p_i g}, \quad z_i(0) = z_{i0}, \quad (18)$$

for $i = 1, 2, ..., N$, where $B_{e_i} = \mathbf{I}_n$, $B_{p_i} = -\mathbf{I}_n \in \mathbb{R}^{n \times n}$ are the corresponding part of the their original matrix as defined in (4) with respect to the i-th pursuer. The cost functions of the players are

$$J_{p_i g} = \frac{1}{2} z_i(t_f)^\intercal S_{p_i g_f} z_i(t_f) + \frac{1}{2}\int_0^{t_f} [z_i(t)^\intercal Q_{p_i g} z_i(t) + u_{p_i g}(t)^\intercal R_{p_i g} u_{p_i g}(t)] dt \quad (19)$$

and

$$J_{e_i g} = \frac{1}{2} z_i(t_f)^\intercal S_{e_i g_f} z(t_f) + \frac{1}{2}\int_0^{t_f} [z_i(t)^\intercal Q_{e_i g} z_i(t) + u_{e_i g}(t)^\intercal R_{e_i g} u_{e_i g}(t)] dt. \quad (20)$$

for $i = 1, 2, ..., N$. In (19) and (20) weights $S_{e_i g_f} = f_{p_i} I_n$; $S_{p_i g_f} = f_{p_i} I_n$; $Q_{e_i g} = q_{e_i} I_n$; $Q_{p_i g} = q_{p_i} I_n$; $R_{e_i g} = r_{e_i} I_n$ and $R_{p_i g} = r_{p_i} I_n$ are the corresponding parts of (6) and (5). The system dynamics (18) and the players' cost functions (19) and (20) form N different 2-player LQ Games and the Greedy closed-loop Nash strategies for each game can be obtained as follows

$$u_{p_i g}^* = -R_{p_i g}^{-1} B_{p_i g} S_{p_i g} z_i, \quad (21)$$
$$u_{e_i g}^* = -R_{e_i g}^{-1} B_{e_i g} S_{e_i g} z_i; \quad (22)$$

for $i = 1, 2, ..., N$, where $S_{p_i g}$ and $S_{e_i g}$ are the solutions to the following 2-coupled differential Riccati equations

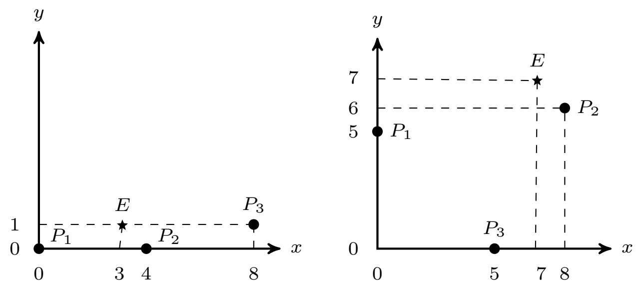
$$\dot{S}_{p_i g} + Q_{p_i g} - S_{p_i g} B_{p_i g} R_{p_i g}^{-1} B_{p_i g}^\intercal S_{p_i g} - S_{p_i g} B_{e_i g} R_{e_i g}^{-1} B_{e_i g}^\intercal S_{e_i g} - S_{e_i g} B_{e_i g} R_{e_i g}^{-1} B_{e_i g}^\intercal S_{p_i g} = 0$$
$$S_{p_i g}(t_f) = S_{p_i g_f} \quad (23)$$

$$\dot{S}_{e_i g} + Q_{e_i g} - S_{e_i g} B_{e_i g} R_{e_i g}^{-1} B_{e_i g}^\intercal S_{e_i g} - S_{e_i g} B_{p_i g} R_{p_i g}^{-1} B_{p_i g}^\intercal S_{p_i g} - S_{p_i g} B_{p_i g} R_{p_i g}^{-1} B_{p_i g}^\intercal S_{e_i g} = 0$$
$$S_{e_i g}(t_f) = S_{e_i g_f}. \quad (24)$$

The evader has N different choices for its strategy and chooses the one that corresponds to the game that gives it most favorable outcome.

IV. Simulation

The main difficulties in implementing any of the strategies mentioned above are the uncertainties faced by each player in computing its own strategy. These uncertainties are due to the fact that in general (1) no player knows what strategy each of the remaining players in the system will be using and (2) no player knows the various parameters and weights in the cost functions of the remaining players. We deal with the first uncertainty by computing the controls, trajectories, and



(a) Scenario 1 (b) Scenario 2

Fig. 4. Initial Position of Players in Simulations

outcome of the game for each of the strategies on one side against all possible strategies on the other side. We deal with the second uncertainty by performing a Monte Carlo analysis on each of the resulting games over the entire parameter space in the cost functions for that game. To illustrate this approach, we consider a simple example of four moving entities on a planar surface ($n = 2$) with one entity as the evader and the other three as pursuers. The evader is trying to escape by maximizing its distance from the pursuers and the pursuers are trying to capture the evader by minimizing these distances. We considered a terminal time $t_f = 3s$ and a capture radius of $d = 0.1$. That is, after 3s, either the evader escapes or it is captured if only one pursuer is at a distance within its capture radius. We consider two different scenarios with the initial positions of the players as provided in Table I and illustrated in Figure 4. To limit the computational costs, we assume that the evader's parameters S_{e_f}, Q_e and R_e are fixed and equal to identity. The parameters of the pursuers are assumed to be uniformly distributed over prescribed ranges as follows: $q_{p_i} \sim U[0,1]$, $f_{p_i} \sim U[0,1]$ and $r_{p_i} \sim U[0,10]$ for $i = 1,2,3$.

In the Team Optimal strategy for the pursuers we considered four possible teaming configurations based on the choices of α_i as described in Table II. When the evader considers a Nash strategy versus a team of pursuers, it should assume that the pursuers have formed a team of specific type as in Table II and this might differ from the actual coefficient that pursuers had chosen for their teaming configuration. This is the reason why we have studied the possibility of different Nash vs Team strategies for the evader and Team Optimal strategies for pursuers.

Each sample of the Monte Carlo analysis has been simulated for all different combinations of strategies for the pursuers and evader. The outcome of each sample is defined to be a Bernoulli random variable of 1 if the pursuers capture the evader with probability of (ρ) and 0 if the evader escapes with probability of $(1-\rho)$. Results of the Monte Carlo simulations are shown in Table III and IV. Based on [21] and [22], to accomplish a 95% confidence level in achieving no more than 1% error in the calculated probabilities, each entry in these tables required $10,000$ runs.. The computations were performed on 10 Intel Xeon 64-bit supercomputer and required 14.9 hours and 35.5 hours of CPU time, respectively.

In interpreting these results, we note that for each evader's

TABLE I
DISTANCES OF PURSUERS TO EVADER IN EACH SCENARIO

	Minimum	Maximum	Average	RMS
Scenario 1	1.41	5	3.19	6.08
Scenario 2	1.41	7.28	5.32	10.39

TABLE II
VALUES OF α_i FOR EACH PURSUER IN DIFFERENT TEAMING CONFIGURATIONS

	Pursuer 1	Pursuer 2	Pursuer 3
Team 1	$0.\bar{3}$	$0.\bar{3}$	$0.\bar{3}$
Team 2	0.8	0.1	0.1
Team 3	0.1	0.8	0.1
Team 4	0.1	0.1	0.8

strategy (each column), the pursuers' preference is to choose a strategy that maximizes the probability of capture. These choices are indicated by ‡. Similarly, for each pursuer's strategy (each row) the evader's preference is to choose a strategy that minimizes the probability of capture. These choices are indicated by †. For the pursuers we define the security strategy as the one that corresponds to the maximum value of the minimum probability in each row (indicated by bold and †). This strategy guarantees the pursuers that the probability of capture will be no less than this value. Similarly, we define the security strategy for the evader as the strategy that corresponds to the minimum value of the maximum probability in each column (indicated by bold and ‡). This strategy guarantees the evader that the probability of capture will be no more than this value.

In Scenario 1 (Table III), the security strategy for the pursuers is All Nash with a probability of capture of 44%, while the security strategy for the evader is Nash vs All with a probability of capture of 57% (i.e. probability of escape of 43%). When these strategies are implemented, the resulting outcome will have a probability of capture of 47% as can be seen in Table III. In Scenario 2 (Table IV), the positions of the players have been changed so only Pursuer 2 is close to

TABLE III
SCENARIO 1: PROBABILITY OF CAPTURE (IN %)

		Evader Strategy					
		Nash vs All	Nash vs Closest	Nash vs Team 1	Nash vs Team 2	Nash vs Team 3	Nash vs Team 4
Pursuers Strategy	All Nash	47	**44**†	62	66‡	57	72‡
	Team 1 Optimal	30	23	18†	31	18†	36
	Team 2 Optimal	**57**‡	30†	97‡	33	68‡	46
	Team 3 Optimal	42	11†	88	17	17	17
	Team 4 Optimal	56	30	95	47	66	21†
	Greedy	28	72‡	54	34	37	15†

TABLE IV
SCENARIO 2: PROBABILITY OF CAPTURE (IN %)

Pursuers Strategy	Evader Strategy: Nash vs All	Nash vs Closest	Nash vs Team 1	Nash vs Team 2	Nash vs Team 3	Nash vs Team 4
All Nash	22‡	20†	28	37‡	26‡	**20‡†**
Team 1 Optimal	8	7	3†	12	6	4
Team 2 Optimal	8	6†	36	7	20	8
Team 3 Optimal	10	6†	73‡	10	16	9
Team 4 Optimal	13	4†	33	7	11	4†
Greedy	3†	50‡	8	3†	16	3†

the evader. Comparing Table III and IV, we notice that the probabilities of capture in Table IV are much lower in general than in Table III. This is expected due to the fact that the average and RMS initial distances to the evader are larger in Scenario 2 than in Scenario 1. In this situation, the security strategy for the pursuers is All Nash with a probability of capture of 20%, and the security strategy for the evader is Nash vs Team 4 with the same probability of capture of 20% (i.e. probability of escape of 80%). We note that the difference between the two scenarios is that if each side uses its security strategy, the outcome will be as expected in Scenario 2 but will not in Scenario 1.

V. CONCLUSION

In this paper, we considered a system of systems approach to the traditional pursuit-evasion game. In this approach the players are subsystems of a large functioning system. When one subsystem decides to split from the rest, a game situation similar to the multi-pursuer one-evader game will result. The separating subsystem (evader) forms its own objective function designed to distance itself away from the remaining subsystems. The conforming subsystems (pursuers), on the other hand, may react in several possible ways. In this paper, we considered three possible ways in which these subsystems may react: First, They may become disorganized non-cooperative units signifying a complete breakdown of the original system into all competitive subsystems. Second, they may organize into cooperative units unified as a team in preventing the separating subsystem from succeeding. And, third, they may disintegrate into fragmented greedy subsystems each acting on its own to prevent the separating unit from succeeding. We formulated and derived necessary conditions for these three strategies in the context of a Linear Quadratic problem. We then performed a Monte Carlo analysis on an illustrative example to show how probabilities of success of each of the strategies can be determined in the presence of uncertainties. These uncertainties refer to the lack of knowledge of the strategy and parameters in the cost function used by the other side. Results show that the pursuers do not always benefit from behaving as a team and that in some cases, acting independently as Nash players assures the highest probability of preventing the evader from escaping no matter what strategy the evader uses.

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